IV-3. THEORETICAL AND PRACTICAL APPLICATIONS OF CAPACITANCE MATRIX TRANSFORMATIONS TO TEM NETWORK DESIGN

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An important aspect of TEM quarter-wave network synthesis and design is the multiplicity of physical configurations that give identical response characteristics. Different network configurations are often required to realize the same basic response for moderate changes in design parameters because of the small range of realizable impedance values. In most design procedures, practical circuit element values are obtained by application of suitable equivalent circuit relationships. In a previous paper [1] a systematic method of obtaining equivalent circuits using transformations of the static capacitance matrix of a parallel coupled line array was introduced and applied to the exact design of interdigital bandpass filters. These transformations can also be used to give a simple physical interpretation to familiar transmission line identities and can be applied to the design of many TEM microwave devices.

As an example of the equivalent circuits obtainable by application of the capacitance matrix transformation, consider the familiar single-section four-port parallel line directional coupler shown in Figure 1(a). The matched constraint requires that $Z_o = \sqrt{Z_{oe}Z_{oo}}$, where Z_{oe} and Z_{oo} are the even and odd mode impedances and Z_o is the characteristic impedance of the terminating lines [2]. The normalized [1] even and odd mode static capacitance values (c) are related to the even and odd mode impedances [2] by

$$Z_{oe} = \frac{376.7}{\sqrt{\epsilon_r}\, c_{oe}} \quad \text{and} \quad Z_{oo} = \frac{376.7}{\sqrt{\epsilon_r}\, c_{oo}} . \tag{1}$$

The corresponding self and mutual static capacitance values are then those given in Figure 1(b). The circuit of Figure 1(b) can be considered as a degenerate three-line network above a ground plane in which the center line is coupled to both outer lines, but is decoupled from ground as shown in Figure 1(c). The electrical performance of this network at ports 1 through 4 is identical to those of the two-line network of Figure 1(b). The capacitance matrix for the network of Figure 1(c) is:

$$\begin{bmatrix} c_{oo} & -(c_{oo} - c_{oe}) & 0 \\ -(c_{oo} - c_{oe}) & 2(c_{oo} - c_{oe}) & -(c_{oo} - c_{oe}) \\ 0 & -(c_{oo} - c_{oe}) & c_{oo} \end{bmatrix} \begin{matrix} \\ \leftarrow n' \\ \\ \end{matrix} \quad \begin{matrix} \\ \uparrow \\ n' \end{matrix} \tag{2}$$

where the diagonal terms are equal to the sum of the capacitances connected to each node and the off diagonal terms are the negative of the capacitance between nodes. Note the assumption that there is no direct coupling between nonadjacent lines; i.e., $c_{13} = c_{31} = 0$. If the center row and column of this matrix is multiplied by an admissable [1, 3] constant (n'), the electrical performance at ports 1 through 4 is unchanged. The effect of a partial capacitance matrix transformation on the corresponding physical realization is shown in Figure 1(d). A complete transformation results when the multiplying constant is chosen such that the outer lines are decoupled from the ground plane. This requires the sum of the elements of the first row or column of the capacitance matrix to be zero. Therefore,

$$c_{oo} - n' (c_{oo} - c_{oe}) = 0 \quad \text{or} \quad n' = \frac{c_{oo}}{c_{oo} - c_{oe}}. \tag{3}$$

The completely transformed network capacitance values are shown in Figure 1(e) and a physical realization with appropriate impedance values is shown in Figure 1(f). The transformed network is seen to be the re-entrant section described by Cohn. [4] The two-line directional coupler and the re-entrant coupled section are limiting cases of a more general three-line network. The two-line network results from a transformation that decouples the center line from the ground plane and the re-entrant section from a transformation that decouples the outer lines from the ground plane. Electrical performance of both networks, and those in which all three lines are coupled to the ground plane, is identical if obtained by using the capacitance matrix transformation described.

The capacitance matrix approach can also be used to derive Kuroda's identities [5] , and provides a simple physical interpretation to these important equivalent network relationships.

As a practical application of the capacitance matrix transformation, the design and construction of a 3:1 bandwidth filter-transformer will be described. The filter-transformer design is based on the interdigital filter prototype and makes use of element value tables [6] computed for interdigital bandpass filters. The normalized static capacitance values for a four-section 0.01 db ripple 3:1 bandwidth interdigital filter with open-circuited terminating lines are shown in Figure 2(a). The prototype incorporates coaxial terminating lines as described in Reference 1. A filter-transformer can be obtained by transformation of the static capacitance matrix such that the capacitance from the second node to ground is eliminated. This is accomplished by multiplying the third and fourth rows of the static capacitance matrix by suitable constants. The resultant static capacitance network is shown in Figure 2(b) and gives a termination ratio of 3.06:1. The capacitance matrix transformation is identical to application of Kuroda's identities to the corresponding S-plane equivalent circuit and is especially convenient (and systematic) for networks containing many elements. ($S = j \tan \pi f/2f_0$ is the frequency transformation introduced by Richards[7] that allows distributed elements to be treated as lumped elements.)

An S-plane equivalent circuit corresponding to the capacitance network of Figure 2(b) is shown in Figure 3(a). The S-plane elements have been normalized to 50 ohms. A schematic cross-sectional drawing showing the location of the circuit elements in a multiple re-entrant coaxial realization is given in Figure 3(b). Symbols have been added to important junction points in Figure 3(a) and Figure 3(b) to aid in visualizing the relationship between the prototype network and its physical realization.

Using the above configuration, a trial network was constructed. The measured and computed transformer performance characteristics are shown in Figure 4, and a photograph of the disassembled unit is shown in Figure 5. The device has the insertion loss characteristic of a four-section 3:1 bandwidth interdigital filter and acts as a 3.06:1 impedance transformer in the passband. The performance of the transformer is comparable to that of a conventional cascade of four quarter-wave elements[6,8] in a network one fourth as long as the stepped impedance design. Because of the coaxial construction, the low impedance terminal of the transformer is also a region of high current density, as required in some applications of broadband transformers. Broader bandwidths and/or larger impedance ratios can be obtained by utilizing more sections as with conventional quarter-wave transformers. The filter-transformer described will be of particular value at UHF and low microwave frequencies because of its compactness.

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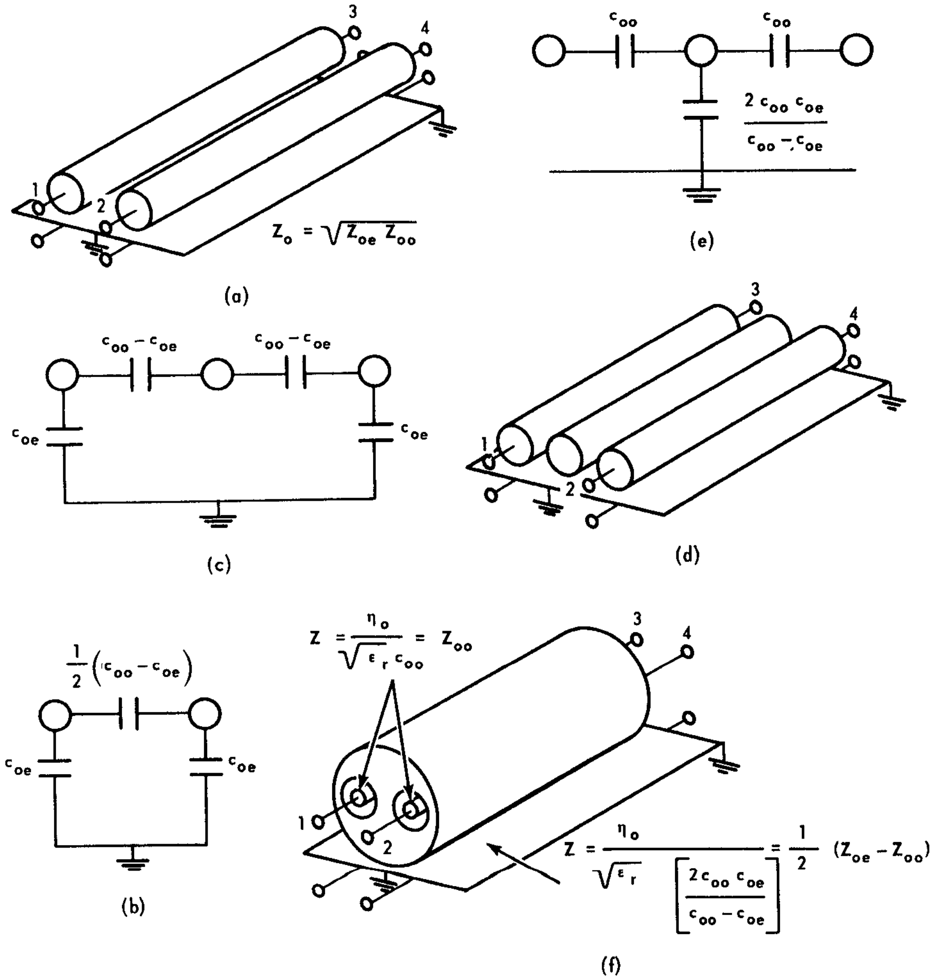
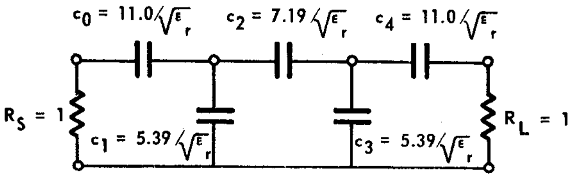
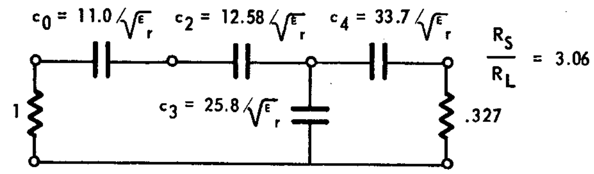


Figure 1. Directional Coupler Equivalent Circuits Obtained by Capacitance Matrix Transformations

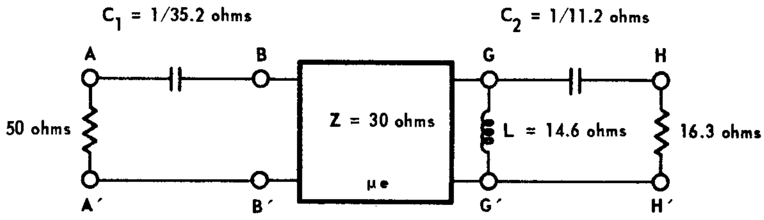


(a) Normalized Static Capacitance Values

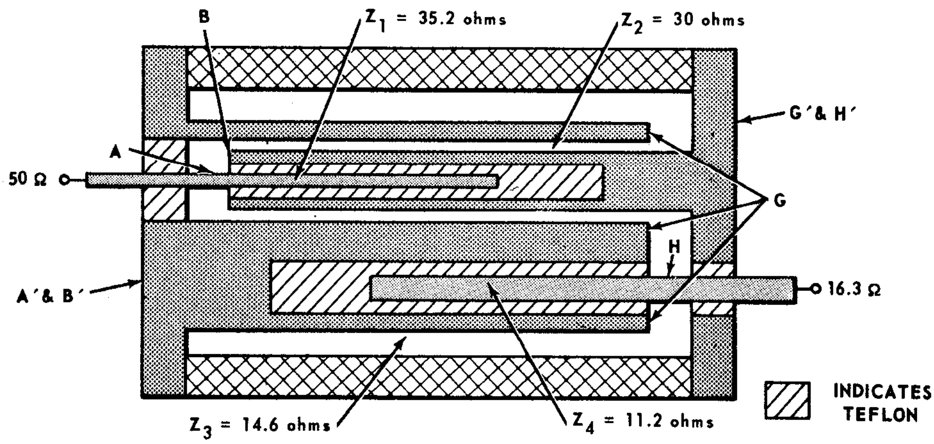


(b) Resultant Network After Transformation of the Capacitance Matrix

Figure 2. Coaxial Filter-Transformer Design



(a) S-Plane Prototype Network Normalized to 50 ohms



(b) Schematic Cross-Sectional Drawing

Figure 3. Coaxial Filter-Transformer

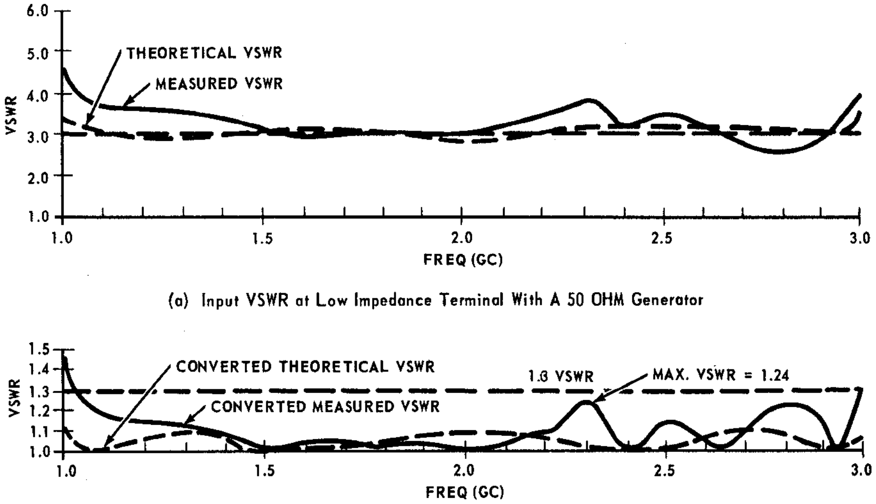


Figure 4. Theoretical and Measured Performance of 3:1 Bandwidth Filter-Transformer

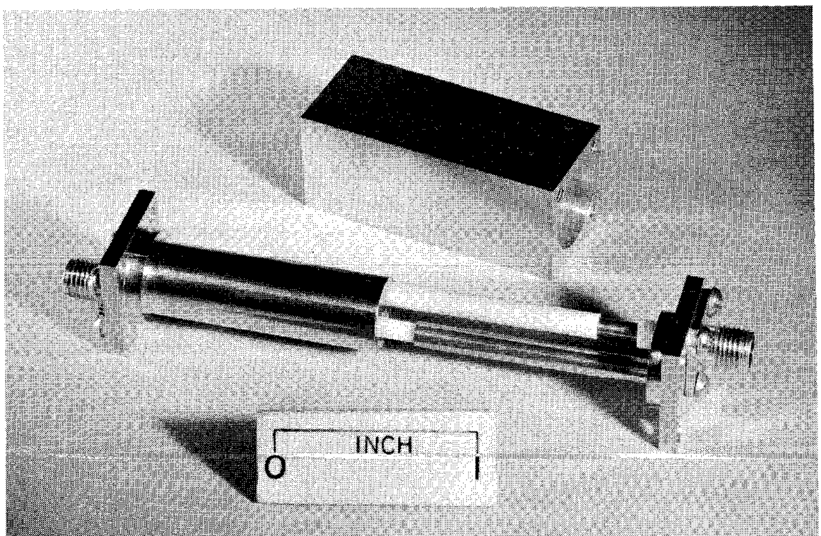


Figure 5. Four-Section Coaxial Filter-Transformer